The use of the F-function of the two-parameter cubic equation of state in vapour-liquid equilibrium **calculations of binary mixtures**

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Abstract

The recently developed F-function of Shaw and Lielmczs, Mak and Lielmezs. and Liu. Lim and Lielmezs, modified to fit the van der Waals type cubic equations of state, has been used to assess the vapoirr-liquid equilibria calculations of binary mixtures over a wide range of *P-T-x-y* values.

Sets of coefficients for use in the binary interaction parameter function $L_{ij} = L_{ij}(T, P, x_i)$ were determined for 23 representative binary mixtures. The prediction (this work) of the vapour-liquid equilibria (bubble point calculations) of the binary systems considered are in excellent agreement with the experimental data, and show slight improvement over the values obtained by means of the original Redlich-Kwong-Soave (RKS) and Peng-Robinson (PR) equations.

LIST OF SYMBOLS

parameters of cubic equation of state
dimensionless parameters (eqn. (A5))
coefficient of F -function
dimensionless coefficients of eqn. (7)
F -function
<i>F</i> -function modification of RKS equation of state
FRKS equation with L_{ii} -function (eqn. (7))
F -function modification of PR equation of state
FPR equation with L_{ii} -function (eqn. (7))
binary interaction parameter for RKS and PR equations
binary interaction parameter for FRKS and FPR equations
vaporization equilibrium ratio

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Subscripts

INTRODUCTION

Recently, Liu et al. [1] extended the work of Lielmezs and co-workers [2,3] by testing the predictive accuracy of the F -function-modified Redlich-Kwong-Soave [4] (FRKS, F-RKS) and Peng-Robinson [5] (FPR, F-PR) equations of state in calculations of $P-V-\overline{T}$ properties of binary mixtures. In this work we test the FRKS, F-RKS, FPR and F-PR equations further by including the vapour-liquid equilibria (bubble point and K value) calculations ot binary systems.

The testing was done in two ways: firstly, by means of fixed isothermal and fixed averaged optimum binary interaction parameters L_u independent of thermodynamic state (FRKS and FPR equations, Tables 1-4, 6), and secondly, following Lielmezs $[6, 7]$ work by introducing instead of the fixed optimum interaction parameter L_{ij} , a binary interaction parameter L_{ij} function of state, $L_{ij}(T, P, x_i)$ (Tables 5, 6, F-RKS, F-PR). To do this testing, experimental data from twenty-nine binary mixtures were selected from a number of sources (Tables $1-4$). These literature data were considered of sufficient reliability; therefore, no further evaluation of their accuracy was made. The total set of experimental data was divided into three main groups of binary mixtures: paraffinic-paraffinic, $CO₂$ -paraffinic and H_2S -paraffinic, and CO_2 -polar and polar-polar compounds. The constants necessary to perform vapour-liquid equilibrium calculations, the T_c , P_r and acentric factor ω values and the required coefficients of the F -function for pure compounds, were taken from the work of Liu et al. [1]. The bubble-point pressures (Tables $1-3$, 6) and vaporization equilibrium ratio K values (Figs. 1-3) were obtained following the general comments of

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calculation methods as given by Reid et al. [8], subject to the derived relations of this work (FRKS, F-RKS, FPR, F-PR) listed in the Appendix.

RESUI.TS AND DISCUSSION

The F-functions for the FRKS, F-RKS, FPR and F-PR equations were determined by methods already outlined by Lielmezs and co-workers [1-3, $6, 7$. Tables $1-3$ compare the results of bubble-point pressure and vapour mole-fraction calculations made by means of fixed isothermal optimum binary interaction parameter k_{ij} and L_{ii} values; Table 4 lists the fixed averaged optimum binary interaction parameter k_{ij} and L_{ij} values optimized with respect to the total number of data points; Table 5 presents the coefficients of the binary interaction parameter $L₀$ -function (eqn. (7)) while Table 6 summarizes the results of bubble point calculations using different methods. Figures 1-3 compare calculated (F-RKS equation) and experimental vaporization equilibrium ratio \tilde{K} values taken at several isotherms for a set of binary mixtures.

The accuracy of the bubble-point pressure and vapour mole-fraction calculations was determined by means of the relative deviation of pressure ΔP (%) and the absolute deviation of vapour mole-fraction Δy , defined as

$$
\Delta P\left(\% \right) = \frac{1}{N} \sum_{i} \left| \frac{P_{\text{cat}} - P_{\text{exp}}}{P_{\text{exp}}} \right|_{i} \times 100 \tag{1}
$$

$$
\Delta y = \frac{1}{N} \sum_{i} |y_{\text{cal}} - y_{\text{expl}}| \times 100
$$
 (2)

where N is the number of data points, and subscripts cal and exp represent calculated and experimental values, respectively.

Following the work of Liu et al. [1], the mixing rules for the FRKS and FPR equations are given as

$$
F_{\rm m} = \frac{1}{b_{\rm m}} \sum_{i} \sum_{j} x_{i} x_{j} (b_{i} b_{j} F_{i} F_{j})^{1/2} L_{ij}
$$
\n
$$
b_{\rm m} = \sum_{i} b_{i} x_{i} \tag{3}
$$

For the original RKS and PR equations of state, the conventional mixing rules have been used

$$
a_m = \sum_i \sum_j x_i x_j (a_i a_j)^{1/2} (1 - k_{ij})
$$
 (5)

where b_m is the same as in eqn. (4).

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TABLE 1

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 \mathcal{L}_{max} . The second state \mathcal{L}_{max}

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 $\sim 10^{-1}$ k

 $\sim 10^{-10}$

TABLE 3

 $\mathcal{F}_{\mathcal{A},\mathcal{C}}$

 $\label{eq:2} \begin{split} \mathcal{L}_{\text{max}}(\mathbf{r}) = \mathcal{L}_{\text{max}}(\mathbf{r}) \mathcal{L}_{\text{max}}(\mathbf{r}) = \mathcal{L}_{\text{max}}(\mathbf{r}) \mathcal{L}_{\text{max}}(\mathbf{r}) \mathcal{L}_{\text{max}}(\mathbf{r}) \mathcal{L}_{\text{max}}(\mathbf{r}) \mathcal{L}_{\text{max}}(\mathbf{r}) \mathcal{L}_{\text{max}}(\mathbf{r}) \mathcal{L}_{\text{max}}(\mathbf{r}) \mathcal{L}_{\text{max}}(\mathbf{r}) \mathcal{L}_{\text{max}}(\mathbf{r}) \math$

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Coefficients of function $L_{12} = e_{12} + f_{12}x_1 + g_{12}p + h_{12}T$ (eqn. (7)) for use in the F-RKS and F-PR methods

 $h_{12}\times 10^3$ 0.38543 -0.36162
 -1.58562 -1.44813 -0.41566 -0.78555 0.04507 -0.19589
 -0.23882 0.15039 $-1,40352$ -0.10911 0.37923 0.35899
0.09062 0.26144 -0.38812 0.37775 -0.21664 -0.63041 -0.96250 -0.79382 0.21381 $g_{\rm i 2} \times 10^2$ $\begin{array}{r} -0.21117 \\ -0.28754 \\ -0.04681 \\ 0.02649 \\ 0.02649 \\ 0.15840 \end{array}$ 0.00760 -0.19177
 -0.04484 0.04621
0.03203 -0.00310 -0.02507 -0.01539 -0.07412 -0.21443
0.09331
0.07749 0.0938 0.10711 -0.26869 0.02559 0.05932 0.04344 0.10645
0.26877 0.48380
0.03852
0.46668
0.77480 -1.37582
0.51239
0.06753
0.36019 -3.50149 -0.59244 -0.32886 0.55256 0.57304 -0.23852 -0.21499 -3.17907 0.10874 -0.16432 0.39431 0.81274 $f_{12} \times 10$ 0.92289
0.96527
0.86123 **C60bD** .31978 1:515 .03412 .38080 1.96857 1.02469
0.77907 0.84399 0.83843 1,93107 38279 32232 24851
18077
16872. .03910
33853 04524 16721 F-PR e_{12} 0.05316
0.04047
0.22517
0.12984
-1.73314
-0.38540 $h_{12}\times 10^3$ -0.31311 -1.60383
 -1.90599 -0.60808 0.04837 -0.75695 0.14198 0.31914 0.47287 -1.55849 0.06888 0.99842 0.34327 0.02457 -0.25613 -0.07166 $g_{12} \times 10^2$ -0.20540 -0.28337 -0.08200 0.22034 0.00909 $\frac{0.05872}{-0.00182}$ 0.24386 -0.25168
 -0.03250 0.05582 -0.02997 -0.04192 -0.17020
 -0.02551 0.05690 0.04847 0.01315 0.04964 0.26485 0.04535 0.11352 3,9086
1.38822 5,83196
0.82969 -0.15791
0.48280 5.78813 0.00245 -3.09380 0.45950 0.62604 -1.17616 0.53200
0.09247
0.47801 0.83496 -0.04394 -0.10500 0.25123 -0.61831 0.36839 1.04788 -0.15261 $f_{12} \times 10$ 0.84623
0.83952
0.83806 F-RKS 1.37259
1.72721
1.98177 0.95863 0,89365 .06518 **BOTHO** 0.97034 1.06985 19264 06198
18198
13138 191965 40601 50654 133681 0.82405 .18228 25692 e_{12} Acetone-methanol $CO₂$ -methanol Acetone-H₂O Methane-H₂O Acetone-CO₂ Ethanol-H₂O $CH_{\mathbf{t}}\text{-}nCH_{\mathsf{10}}$ $CH_{\mathbf{q}}$ -n $\mathbf{C}_6\mathbf{H}_{\mathbf{p}}$ $CO₂-nCH₁₀$ $H_2S-nC_5H_{12}$ $CO₂-nC₆H₁₄$ $CH₄-CH₆$ $CH₄CH₈$ $H_2S-C_3H_8$ **O**_rH_rHz System

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 $\frac{1}{2}$

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Fig. 1. Comparison of calculated (F-RKS) and experimental K values for (left) ethane- n pentane at 344.3, 377.6 and 410.9 K [17]; and (right) methane-propane at 277.6, 310.9, 327.6 and 344.3 K [10].

Fixed binary interaction parameter L_{ii} values

The fixed optimum isothermal binary interaction parameter L_{ij} values for seventy-eight isotherms of the twenty-nine binary systems were calculated from the FRKS and FRP equations by minimizing the objective function SP

Fig. 2. Comparison of calculated (F-RKS) and experimental K values for (left) carbon dioxide-n-hexane at 313.15,353.15 and 393.15 K [25]; and (right) acetone-carbon dioxide at 298.15 and 313.15 K [31].

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Fig. 3. Comparison of calculated $(F-RKS)$ and experimental K values for (left) methanolwater at 373.15, 423.15, 473.15 and 523.15 K [32]; and (right) ethanol-water at 423.15, 473.15, 523.15, 548.15 and 573.15 K [33].

given as

$$
SP = \frac{100}{N} \sum_{i}^{N} \left(\left| \frac{P_{\text{cal}} - P_{\text{exp}}}{P_{\text{exp}}} \right|_{i} + |y_{\text{cal}} - y_{\text{exp}}|_{i} \right)
$$
(6)

over the same set of experimental data. The optimum k_{ij} values for the original RKS and PR equations were obtained using the same optimum search procedure. Tables 1-3 compare all the results. Table 1 shows the relative deviations of pressure ΔP (%) and absolute deviations of vapour mole-fraction Δy for eleven paraffin-paraffin compound binary systems. Table 2 compares the same (ΔP (%) and Δy) for seven CO₂-paraffin and four H₂S-paraffin compound binary mixtures. Table 3 shows the ΔP (%) and Δy comparison for seven $CO₂-polar$ and polar-polar compound binary systems. As seen from Tables 1–3 (fixed isothermal L_{ij} and k_{ij} values) for paraffin-paraffin, CO_2 -paraffin and H_2S -paraffin compound binary systems, the original RKS and PR equations of state have a slight edge over the FRKS and FPR equations. For $CO₂-polar$ and polar-polar compound binary systems, the FRKS and FPR equations show better prediction patterns than the corresponding RKS and PR equations.

Interaction parameter function $L_i(T, P, x_i)$.

Following Lielmezs [6,7] work, the binary interaction parameter function $L_{ij}(T, P, x_i)$ has been introduced into the FRKS and FPR equations, yielding the F-RKS and F-PR equations for vapour-liquid equilibrium calculations. We write the $L_y(T, P, x_i)$ function as

$$
L_{ij} = e_{ij} + f_{ij}x_i + g_{ij}P + h_{ij}T
$$
 (7)

where coefficients e_{ij} , f_{ij} , g_{ij} and h_{ij} are characteristic dimensionless [6, 7] constants of the given binary systems.

To satisfy the requirement that $L_{ii} = 1$ and $L_{ii} = L_{ii}$, we set

$$
e_{ij} = 1, \t f_{ii} = 0, \t g_{ii} = 0, \t h_{ii} = 0
$$

$$
e_{ji} = e_{ij}, \t f_{ji} = \frac{x_i}{x_j} f_{ij}, \t g_{ji} = g_{ij}, \t h_{ji} = h_{ij}
$$
 (8)

The coefficients e_{ij} , f_{ij} , g_{ij} and h_{ij} (eqn. (7)) for use in the F-RKS and F-PR equations were determined as follows. First, the optimum L_{ij} value (eqn. (7)) for every experimental data point of a given binary system was calculated by minimizing the objective function SP in conjunction with the F-RKS and F-PR equations (eqn. (6), $i = 1$). Those experimental data points which gave sharply fluctuating L_{ij} values were discarded, thus yielding a directional set of L_{ij} values for the given binary system. Then, from this stable set of L_y values, values of the coefficients e_{ij} , f_{ij} , g_{ij} and h_{ij} , again in conjunction with the F-RKS and F-PR equations, were obtained by means of multi-property linear regression methods. The calculated values of the coefficients e_{ij} , f_{ij} , g_{ij} and h_{ij} (eqn. (7)) for the twenty-three binary systems are listed in Table 5.

To calculate the bubble-point pressure using the L_{θ} -function (eqn. (7)) for the F-RKS and F-PR equations, an initial \overline{P} value is first assumed. For this initial P, using eqn. (7) an initial L_{ij} value can be obtained and the iteration process started, i.e. when the pressure P value of the first iteration has been obtained, a new L_{ij} value can be calculated from it. The iterations need to be repeated until a preset convergence limit has been reached. In this work the limit is

$$
\left|1-\sum_{i} y_i\right| \leq 10^{-5} \qquad |(P_n-P_{n+1})|/P_{n+1} \leq 10^{-4}
$$

Table 6 presents a comparative evaluation of the bubble point calculations performed by means of the averaged optimum values of the fixed binary interaction parameters L_{ij} and k_{ij} (FRKS and FPR, and RKS and PR equations) and the binary interaction parameter L_{ij} -function (F-RKS and F-PR equations).

Table 6 shows that over narrower T and P ranges, the FRKS and FPR equations (fixed, state-independent binary interaction parameter L_{ij} values) predict vapor-liquid equilibria with reasonable accuracy. For larger T and P ranges, for instance, $CH_4\neg nC_4H_{10}$, $CH_4\neg nC_6H_{14}$, $CH_4\neg nC_8H_{18}$ binaries, the predictive accuracy of the FRKS and FPR equations decreases. However, the F-RKS and F-PR equations (state-dependent binary interaction parameter L_{μ} -function, eqn. (7)) predict the vapour-liquid equilibria with high accuracy for all the systems tested. Figures $1-3$ confirm further the predictive superiority of the F-RKS and F-PR equations. These figures

 $\omega_{\rm LSR} \sim 10$

show vaporization equilibrium ratio K values plotted against the pressure P at several fixed temperatures. As seen from Figs. 1-3, the F-RKS equation predicts accurately the vapourization equihbrium ratio K values for a set of selected non-polar and polar binary systems up to their critical points.

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The generalized F-function modification of the two-parameter cubic equation of state $[1-3]$ can be written as

$$
P = \frac{RT}{V - b} - \frac{\Omega_a}{\Omega_b} \frac{RTb}{V^2 + ubV + wb^2} F(T_r)
$$
 (A1)

where u and w must satisfy the constaints

$$
w > -u - 1 \qquad \text{for } u \ge -2
$$

\n
$$
w > \frac{u^2}{4} \qquad \text{for } u \le -2
$$
 (A2)

while

$$
b = \Omega_{\rm b} \frac{RT_{\rm c}}{P_{\rm c}} \tag{A3}
$$

$$
F(T_r) = \sum_{n=1}^{m} c_n T_r^{(1-n)/2}
$$
 (A4)

where T_r is reduced temperature and c_n ($n = 1, 2, \ldots, m$) are empirical substance-dependent coefficients.

The compressibility factor equation is

$$
Z3 + (uB - B - 1)Z2 + (A + wB2 - uB - uB2)Z
$$

– (AB + wB² + wB³) = 0 (A5)

where

$$
A = \frac{\Omega_n}{\Omega_b} BF
$$
\n
$$
B = \frac{bP}{RT}
$$
\n(A6)

The generalized fugacity coefficients of component k in a mixture is

$$
\ln \phi_k = \frac{b_k}{b_m} (Z - 1) - \ln(Z - B) + \frac{\Omega_a}{\Omega_b} \frac{F_m}{b_m \sqrt{u^2 - 4w}}
$$

$$
\times \left[b_k - \frac{2(b_k F_k)^{1/2} \sum_i x_i (b_i F_i)^{1/2} L_{ki}}{F_m} \right]
$$

$$
\times \ln \left[\frac{2Z + B(u + \sqrt{u^2 - 4w})}{2Z + B(u - \sqrt{u^2 - 4w})} \right]
$$
(A8)

The F-function modification of the Redlich–Kwong–Soave equation of

 $\label{eq:2.1} \mathcal{L}=\mathcal{L}^{\text{max}}_{\text{max}}\left(\mathcal{L}^{\text{max}}_{\text{max}}\right) \left(\mathcal{L}^{\text{max}}_{\text{max}}\right) \left(\mathcal{L}^{\text{max}}_{\text{max}}\right)$

state (FRKS):
\nu = 1, w = 0
\n
$$
\Omega_a = 0.42748
$$

\n $\Omega_b = 0.08664$
\n $Z^3 - Z^2 + (A - B - B^2)Z - AB = 0$
\n $\ln \phi_k = \frac{b_k}{b_m} (Z - 1) - \ln(Z - B) + \frac{\Omega_a}{\Omega_b} \frac{F_m}{b_m}$
\n $\times \left[b_k - \frac{2(b_k F_k)^{1/2} \sum_i x_i (b_i F_i)^{1/2} L_{ki}}{F_m} \right]$
\n $\times \ln(1 + \frac{B}{Z})$ (A10)

The F -function modification of the Peng–Robinson equation of state (FPR):

$$
u = 2, \t w = -1
$$

\n
$$
\Omega_{a} = 0.45724
$$

\n
$$
\Omega_{b} = 0.07780
$$

\n
$$
Z^{3} + (B - 1)Z^{2} + (A - 2B - 3B^{2})Z - (AB - B^{2} - B^{3}) = 0
$$
 (A11)
\n
$$
\ln \phi_{k} = \frac{b_{k}}{b_{m}}(Z - 1) - \ln(Z - B) + \frac{\Omega_{a}}{\Omega_{b}} \frac{F_{m}}{2\sqrt{2}} \frac{F_{m}}{b_{m}}
$$

\n
$$
\times \left[b_{k} - \frac{2(b_{k}F_{k})^{1/2} \sum x_{i}(b_{i}F_{i})^{1/2}L_{ki}}{F_{m}} \right]
$$

\n
$$
\times \ln \left[\frac{Z + B(1 + \sqrt{2})}{Z + B(1 + \sqrt{2})} \right]
$$
 (A12)

 $\label{eq:2.1} \frac{1}{\left(1-\frac{1}{2}\right)}\left(\frac{1}{\left(1-\frac{1}{2}\right)}\right)^{\frac{1}{2}}\left(\frac{1}{\left(1-\frac{1}{2}\right)}\right)^{\frac{1}{2}}\left(\frac{1}{\left(1-\frac{1}{2}\right)}\right)^{\frac{1}{2}}\left(\frac{1}{\left(1-\frac{1}{2}\right)}\right)^{\frac{1}{2}}\left(\frac{1}{\left(1-\frac{1}{2}\right)}\right)^{\frac{1}{2}}\left(\frac{1}{\left(1-\frac{1}{2}\right)}\right)^{\frac{1}{2}}\left(\frac{1}{\left(1-\frac$

 $\label{eq:2} \frac{1}{\sqrt{2}}\int_{0}^{\infty}\frac{1}{\sqrt{2\pi}}\left(\frac{1}{\sqrt{2\pi}}\right)^{2}d\mu_{\rm{max}}\,,$

 $\mathcal{L}^{\mathcal{L}}(\mathcal{L}^{\mathcal{L}})$. The contribution of the contribution of $\mathcal{L}^{\mathcal{L}}$

 $\label{eq:2} \frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\right)\right)^{2} \left(\frac{1}{2}\left(\frac{1}{2}\right)\right)^{2} \left(\frac{1}{2}\right)^{2} \left(\frac{$

 $\label{eq:2} \mathcal{L}_{\frac{1}{2}}\left(\mathcal{L}_{\frac{1}{2}}\right) = \mathcal{L}_{\frac{1}{2}}\left(\mathcal{L}_{\frac{1}{2}}\right) = \mathcal{L}_{\frac{1}{2}}\left(\mathcal{L}_{\frac{1}{2}}\right)$